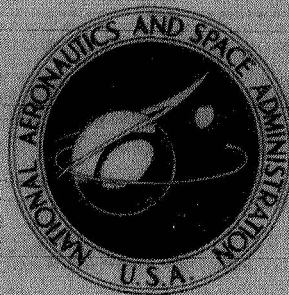


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**EFFECT OF LOW THRUST AND EARTH
OBLATENESS ORBIT PERTURBATIONS
ON THE DETERMINATION OF THE
LAUNCH WINDOW FOR THE SERT II MISSION**

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Cleveland, Ohio 44135

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SUMMARY

The SERT II (Space Electric Rocket Test) mission objective is to provide a long-term evaluation of the performance and reliability of an ion thruster system in the space environment. Presented in this report is a review of the procedures employed to determine the launch window for the SERT II mission. Background is given relating to the Earth-oblateness, Sun-synchronous technique for achieving the continuous sunlight necessary to provide solar electric power to the thruster system. The equations used to mathematically model the oblateness perturbations and low-thrust continuous orbit altitude changing are stated.

A launch window is found such that either thruster system can be operated continuously for the first 6 months. Within this window, consideration is given to selecting a launch time such that the other thruster system can then be operated for a discontinuous period of 6 months. Also included in the selection of the launch time is the Sun interference problem associated with the guidance system of the launch vehicle.

Data for the February 3, 1970, launch date are presented, showing the application of these launch constraints and the tradeoffs considered in arriving at the launch time of 1850 P.s.t.

INTRODUCTION

The SERT II (Space Electric Rocket Test) spacecraft was launched from the Western Test Range on February 3, 1970, at 1849:50 P.s.t. and injected into a 540-nautical-mile (1000-km) near-polar, circular orbit. The primary objective of the SERT II flight is to provide a long-term evaluation of the performance and reliability of an ion thruster

system while in the space environment. Because the electrical power for continuous operation of the thruster is provided by a solar array, a trajectory which is in continuous sunlight for a time sufficient to accomplish the mission objectives is required.

For a low-altitude orbit to remain in continuous sunlight, it must be oriented so that the orbit plane is nearly perpendicular to the Earth-sunline throughout the mission. Injecting the spacecraft into a near-polar, "sunrise-sunset" orbit satisfies this requirement at the start of the mission. However, as the Sun moves along the ecliptic during the year, the orbit plane must revolve at nearly the same rate to maintain its position normal to the sunline. The phenomenon of Earth oblateness can be employed to cause the orbit plane to precess at the desired rate if the altitude and inclination of the orbit are properly selected. The theory of this precession has been treated in references 1 and 2.

The SERT II spacecraft is also subjected to low-magnitude, continuous thrust. This results in a continuous change in orbital altitude, which in turn produces a varying orbital precession rate. An analysis of low-thrust orbit raising in continuous sunlight has been presented in reference 3. Based upon this analysis, this report presents the techniques employed in selecting the launch time which satisfies the SERT II mission requirement. Included are the effects upon the orbital precession of orbital changes due to thrusting, thruster failures, and orbital injection errors. The launch window thus obtained is then further constrained by optimization of the total mission time of two separate continuous sunlight periods and by a Sun interference problem associated with the guidance system of the launch vehicle.

SYMBOLS

a	thrust-weight ratio
ET	equation of time correction, hr
G. m. t.	Greenwich mean time
H	orbit altitude above equatorial radius, nautical miles; km
HA	hour angle of ascending node at time of equatorial crossing, hr
i	orbit inclination, deg
P. s. t.	Pacific standard time
R	equatorial radius, nautical miles; km
T	time of launch so that first ascending nodal crossing occurs at sunrise
t	time, hr

Δt	mission time, days
α	right ascension of Sun, deg
δ	declination of Sun, deg
η	angle between orbit perpendicular and Earth-sunline, deg
τ	time of sunrise, hr
ψ	angle by which orbit perpendicular lags Earth-sunline in longitude, deg
Ω	right ascension of orbit perpendicular, deg
$\Delta\Omega$	change in Ω in Δt

Subscripts:

an	ascending node
Lat	local apparent time
Lmt	local mean time
0	initial value

Superscripts:

time rate of change

BACKGROUND

Configuration

The orbiting configuration of the SERT II spacecraft is shown in figure 1. The spacecraft contains two identical thruster systems. One of these is oriented to provide a component of thrust in the direction of the orbital velocity, while the second provides a component in the opposite direction. The spacecraft uses a gravity-gradient, control-moment gyro method of stabilization which tends to orient the z-axis along the local vertical and the y-axis perpendicular to the orbit plane. As shown in figure 1, the solar array is fixed to the spacecraft so that the nominal orientation of the array is in the orbit plane. The problem then of providing continuous power to the thruster reduces to one of assuring that the orbit is in continuous sunlight.

Orbit Selection

The choice of an initial orbit altitude for SERT II was influenced by several factors. First, it was required that the orbit be achieved using the Thorad-Agena launch vehicle. Second, the initial altitude was required to be such that adverse environmental effects would not be obtained over the possible altitude range. This range was bounded by the maximum and minimum altitudes which would be obtained for either the orbit raising or orbit lowering thrust for the mission duration. Thus, the maximum altitude was required to be sufficiently low that adequate gravity-gradient torques would be obtained and that excessive solar array degradation from high-energy protons and electrons would be avoided. On the other hand, the minimum altitude was required to be sufficiently high that excessive aerodynamic drag and torques were not encountered. Upon consideration of all factors, an initial altitude of 540 nautical miles (1000 km) was chosen. To obtain continuous sunlight for this altitude, the orbit plane must be maintained nearly perpendicular to the Earth-sunline throughout the mission. Injecting the spacecraft into a near-polar, "sunrise-sunset" orbit satisfies this requirement at the start of the mission. Figure 2 shows the geometry of the orbit-plane - sunline relation for a sunset launch from the Western Test Range. If the Sun is considered to move along the ecliptic during the year, the orbit plane must revolve at nearly the same rate to maintain its position normal to the sunline.

Earth Oblateness Effect

As discussed in references 1 to 3, the oblateness or excess Earth equatorial mass causes a component of gravitational force out of the orbit plane which reverses its direction at the line of nodes. The gravitation force causes a torque about the line of nodes. The gyroscopic nature of the satellite in its orbit causes this torque to manifest itself in the precession of the orbital plane about the polar axis of the Earth. For the given initial altitude, the orbital inclination can be selected such that the orbital precession matches the Sun's mean rate and is in the direction shown in figure 2. However, the actual position of the Sun deviates from the mean Sun because of the slight eccentricity of the Earth's orbit about the Sun and the obliquity of the ecliptic. And since the precession rate is a function of both altitude and inclination, the injection errors in these parameters will influence the initial precession rate, and the continuous change in orbital altitude due to thrusting will produce a variable precession rate. These factors were included in the mathematical model describing the geometry of the orbit-plane - sunline relation.

Describing Equations

The equations used to simulate both the effect of the ion thruster thrust on the orbital altitude and the nodal precession rate influenced by a variable altitude were derived in reference 3.

The time required to change the orbital altitude from H_0 to H , under the influence of continuous circumferential thrust, is

$$\Delta t = \frac{C_1}{a} \left[(R + H_0)^{-1/2} - (R + H)^{-1/2} \right] \quad (1)$$

where C_1 is a constant equal to 0.549 for U. S. customary units and 0.747 for SI units; a is the thrust-weight ratio; R is the Earth's equatorial radius; and Δt is in days.

The equation describing the oblateness effect or precession of the line of nodes is

$$\dot{\Omega} = -9.96 \left(\frac{R}{R + H} \right)^{7/2} \cos i \quad (2)$$

where $\dot{\Omega}$ is in degrees per day. As a function of the initial and final altitudes, the change in the location of the line of nodes or orbit perpendicular is

$$\Delta \Omega = -C_2 \frac{\cos i}{a} \left[(R + H_0)^{-4} - (R + H)^{-4} \right] \quad (3)$$

where $\Delta \Omega$ is in degrees and C_2 is a constant equal to 1.637×10^{12} for U. S. customary units and 1.926×10^{13} for SI units. The definition of the angle between the orbit perpendicular and the sunline is found from figure 3. For the sunset launch condition, the orbit perpendicular is opposite to the direction of the angular momentum vector. Taking the dot product of the sunline vector and the orbit perpendicular vector gives the angle between the orbit perpendicular and the sunline for sunset launches:

$$\cos \eta = \cos \delta \sin i \cos (\alpha - \Omega) - \sin \delta \cos i \quad (4)$$

With the angle by which the orbit perpendicular lags the sunline in longitude defined by $\psi = \alpha - \Omega$, the angle η is defined as

$$\cos \eta = \cos \delta \sin i \cos \psi - \sin \delta \cos i \quad (5)$$

If the Earth shadow is considered a cylinder, the spacecraft is in continuous sunlight when

$$\cos \eta > \left(\frac{R}{R + H} \right) \quad (6)$$

Thus, given a starting altitude, inclination, launch date, and initial misalignment of the sunline and orbit perpendicular, time histories of the angle η can be found which terminate when inequality (6) is violated.

SOLUTION OF LAUNCH WINDOW

Mission Operation Goals

The primary operation goal of the SERT II flight is to test either the orbit raising or orbit lowering thruster system for a continuous sunlight period of 6 months. The secondary objective is to operate the second thruster system for a discontinuous sunlight period of 6 months. The launch window as defined in the following section satisfies only the primary operation. Within this launch window, an optimum time is found such that the secondary objective is also met.

Launch Window Definition

The problem in finding the launch window on any given date is to determine those values of the initial longitudinal misalignment ψ which put the spacecraft in continuous sunlight for the required time. For any allowable misalignment, the minimum continuous sunlight time must be obtained when the nominal orbit is subjected to injection errors and either the orbit raising or orbit lowering thruster is operated for the entire test period.

Figure 4 shows the Earth-sunline and two orientations of the orbit perpendicular projected onto the equatorial plane at the start of the mission. Momentarily let us assume that the total angle η consists only of the longitudinal misalignment ψ and let us study the relative positions of the sunline and orbit perpendicular as the precession rate is perturbed. Both the orbit perpendicular and the sunline move in an eastward or counterclockwise rotation. For orientation A, an orbit precession rate which is slower than the nominal or Sun-synchronous rate will result in a clockwise rotation of A relative to the sunline. In this case, the allowable orbit-perpendicular - sunline angle will be exceeded earlier than if a faster precession rate were acquired. By a similar argument,

the allowable angle will be exceeded by B sooner for a precession rate faster than the nominal than for a slower precession rate. From equation (2), the orbit lowering thruster causes a faster precession of B. Also, the injection time of the orbit for orientation A is earlier than for orientation B. The opening of the launch window is therefore defined by using a precession rate which is slower than the nominal. The orbit raising thruster is operating for this case. The closing of the launch window is defined by using a precession rate faster than the nominal and operating the orbit lowering thruster. The problem is to find values of ψ_A and ψ_B such that, for the above precession rates, no shadowing occurs for the required time.

However, for launch dates immediately following the solstice, it was found that launch windows determined in this way which produced sunlight times greater than 6 months allowed shadowing at the beginning of the mission, that is, at launch. For these cases, the boundaries of ψ are therefore determined by the initial orbit geometry through the use of equations (5) and (6). The maximum sunline - orbit-perpendicular angle η is determined from equation (6). Therefore, equation (5) determines symmetric values of ψ_A and ψ_B because the function of ψ is even.

Nominal Orbit and Injection Dispersions

Based on preliminary launch window studies, the nominal orbit was chosen to have a 540-nautical-mile (1000 km) altitude, a 99.1° inclination, and a near zero eccentricity.

The three sigma orbital dispersions are presented in reference 4. The orbital altitude dispersions, derived from the orbital period dispersions, together with the inclination dispersions were combined to determine the fast and slow precession rates. The proper combination is evident by referring to equation (2).

Deriving the fast and slow precession rates in this manner resulted in rates which were correspondingly faster and slower than the precession rate dispersions given directly in reference 4. In those cases where the launch window is determined by the initial sunlight condition, the same altitude and inclination dispersions are applied as perturbations to the initial orbit geometry. Therefore, applying only the altitude and inclination errors adequately describes the dispersions in both cases.

Calculation Procedure

A computer program based upon the describing equations and including ellipse data for the Sun was developed for the purpose of calculating the angular boundaries corresponding to the opening and closing of the launch window. Given a launch date and the

orbital dispersions, the program selected initial estimates of ψ_A and ψ_B and iterated so that the final values of ψ_A and ψ_B resulted in continuous sunlight missions of 6 months.

If ψ is defined as the hour angle by which the orbit perpendicular lags the Earth-sunline (noon) in longitude, it can also be defined as the hour angle by which the right ascension of the ascending node lags sunrise, or the hour angle by which the right ascension of the descending node lags sunset. The procedure used to calculate the launch times from the angular boundaries is derived in the appendix.

For the February 3 launch date, the angular boundaries of the launch window were found to be $\psi_A = 15.5^\circ$ and $\psi_B = -15.5^\circ$. Utilizing the procedure outlined in the appendix, the lift-off times corresponding to the opening and closing of the window were 1735 P.s.t. and 1938 P.s.t.

OPTIMIZATION AND CONSTRAINTS DETERMINING LIFT-OFF TIME

Launch Constraints

For guidance through the ascent phase, the horizon sensors on the Agena are scanning on both the left and right sides of the vehicle in the nose forward orientation. The launch time is constrained so that the Sun does not graze the field of view of the operating horizon sensor during and surrounding the powered portions of the ascent. Figure 2 shows that the right head could view the Sun. If the Sun is viewed, the horizon sensor system interprets the Sun as an extension of the infrared Earth horizon, and consequently errors in the pitch and roll attitude information can result. This Sun interference during the pitch down maneuver at the first burn is especially undesirable because of the length of the burn. If the roll attitude signal is in error, the pitch rate will induce a yaw attitude error, resulting in the thrust vector being oriented out of the transfer plane, and hence injection errors will result.

An analysis performed by the launch vehicle contractor showed that within the launch window for February 3 there was no period of time during which the Sun would not interfere with the operation of the horizon sensor. A modification was made to the guidance system such that the right head was disabled from 1 minute after first burn cutoff and throughout the coast period and second burn. Disabling the horizon sensor and hence the gyrocompassing through coast and second burn was shown by an error analysis to have a negligible effect upon the injection errors.

The launch time was then constrained such that there would be no Sun interference during first burn and the 1-minute period following first burn cutoff. To satisfy this constraint, the opening of the launch window for February 3 was delayed until 1817 P.s.t.

One-Year Mission Optimization

For certain launch times within the primary launch window the secondary mission goal (6 months discontinuous operation of the second thruster) can be realized. After 6 months operation of the first thruster, the second thruster will be operated, interrupted by a period of possibly several months when the spacecraft is in shadow.

Profiles of sunlight and shadow times were prepared for times within the 6-month launch window. These profiles included the two thruster operational sequences for each of the two orbits resulting in the fast and slow precession rates. The results for the February 3 launch date are shown in figures 5 to 8 with figures 5 and 6 representing an initial orbit having a fast precession rate. Operating the orbit raising thruster for the first 6 months slows the otherwise fast precession rate so that the excursion of the orbit perpendicular from the Earth-sunline is not as rapid as it would be if the orbit lowering thruster were operating. Although figures 5 and 6 do not yield a definite indication, the result of first operating the compensating thruster (that thruster which reverses the longitudinal excursion of the orbit perpendicular from the sunline) should be to decrease the shadow period and hence increase the second sunlight period. Figures 7 and 8, representing the slow precession case, clearly indicate a better profile when first operating the compensating thruster.

Selection of Launch Time

Prior to launch preparations, retaining the option of starting either the orbit raising or orbit lowering thruster for the first 6 months until after launch seemed desirable. The data for the achieved orbit would then be used to evaluate the sunlight and shadow periods depending on the thruster sequence. Preflight testing of the thruster systems showed a better performance of the system to be used for orbit raising. During the launch preparations, a slow gas leak was discovered in the neutralizer feed system of the orbit raising thruster. Analyses showed that the pressure remaining in the feed system would be acceptable to force the mercury to the neutralizer for the next 6 months. These two factors, especially the latter, caused the selection of the launch time to be based upon operating the orbit raising thruster first.

Referring to the mission profiles, figure 8 shows that the shadow period would be too long for launch times prior to 1850 P.s.t. had this particular orbit been achieved. Referring to figures 5 to 7 shows the shadow period obtained for a launch at 1850 P.s.t. to be acceptable, and all four profiles indicate a comparable first sunlight period. A lift-off time of 1850 P.s.t. for the February 3 launch was therefore recommended, with the final allowable lift-off time as 1938 P.s.t.

CONCLUDING REMARKS

The purpose of this report has been to outline the basic Sun-synchronous orbit theory and procedures used to determine the launch time for the SERT II spacecraft. A simplified set of equations describing the orbital motion was used to determine the boundaries of a launch window assuring 6 months of continuous operation of either the orbit raising or orbit lowering thruster. Optimization within this launch window to obtain a 1-year mission composed of two separate continuous sunlight periods and applying other launch constraints resulted in a 48-minute launch opportunity for the February 3, 1970, launch.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 19, 1970,
120-26.

APPENDIX - DERIVATION OF LIFT-OFF TIME EQUATION

The angular boundaries ψ of the launch window can be interpreted as an hour angle before or after the actual sunrise at the first ascending node following orbit injection.

To calculate the lift-off times, we shall first derive the time of launch such that the spacecraft passes through the ascending node when it is sunrise at that longitude. The launch window or lift-off times can then be found by adding (or subtracting) the times corresponding to the opening and closing hour angles. Consider an observer on the equator at the longitude of the first ascending node past injection. The observer desires the spacecraft to pass overhead at the local apparent time of sunrise τ_{Lat} . The local mean time of sunrise is

$$\tau_{Lmt} = \tau_{Lat} - ET$$

where ET is the equation of time correction between the apparent (actual) Sun and the mean Sun as shown in figure 9. The local mean time of launch referenced to the observer's meridian T_{Lmt} to place the spacecraft overhead at sunrise is

$$T_{Lmt} = \tau_{Lat} - ET - t_{an}$$

where t_{an} is the time to get from launch to the first ascending node past orbit injection. The launch time referenced to Greenwich is

$$T_{G.m.t.} = T_{Lmt} - HA_{an}$$

where HA_{an} is the hour angle corresponding to the east longitude of the first ascending node.

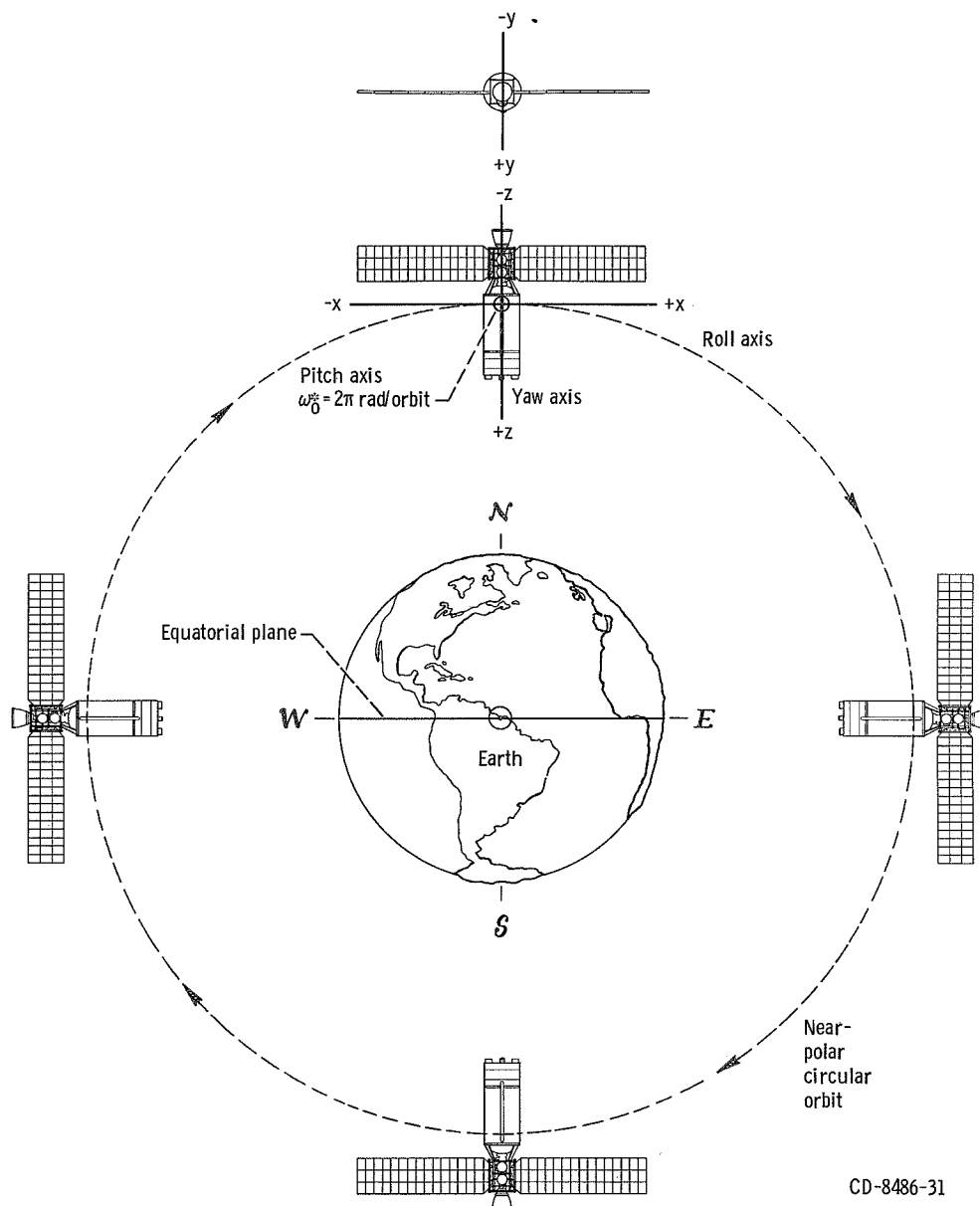
The time $T_{G.m.t.}$ is the launch time to place the spacecraft passing through the first ascending node at sunrise for that longitude. The lift-off times corresponding to the opening and closing hour angles are found by

$$\text{lift-off}_{G.m.t.} = (\tau_{Lat} - ET - t_{an} - HA_{an} - \psi/15^\circ)\text{hr}$$

$$\text{lift-off}_{P.s.t.} = \text{lift-off}_{G.m.t.} - 8 \text{ hr}$$

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CD-8486-31

Figure 1. - SERT II spacecraft-vehicle coordinate system in orbit viewed from Sun for spring launch and sunrise orbit injection.

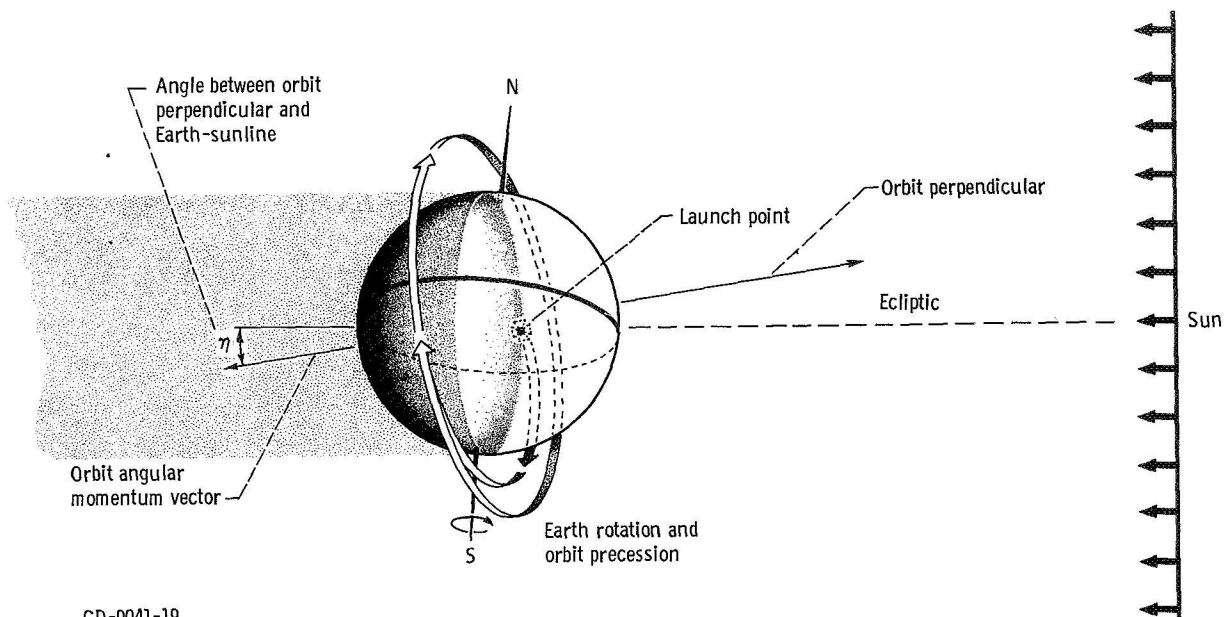


Figure 2. - Orbit-plane - Sun geometry for spring-sunset launch.

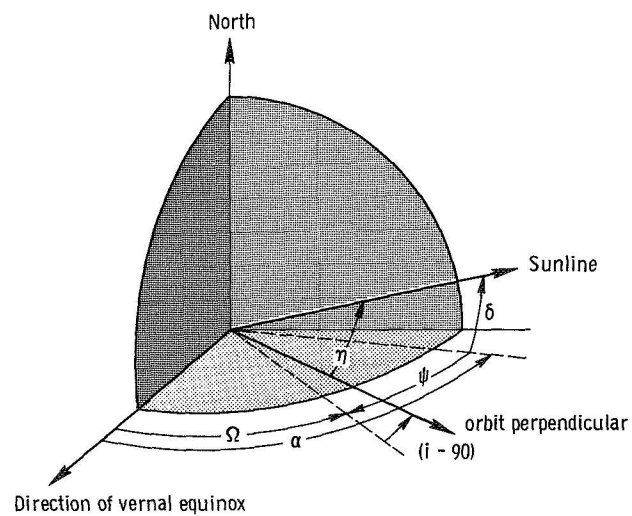


Figure 3. - Angle definition

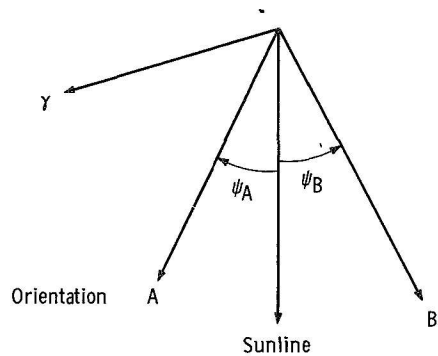


Figure 4. - Possible initial orientations of orbit perpendicular with respect to sunline as projected onto equatorial plane.

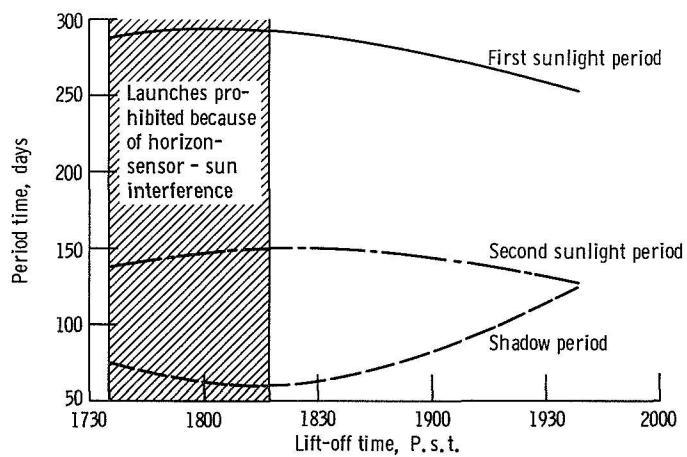


Figure 5. - Mission profile for fast precession rate and first operating orbit lowering thruster.

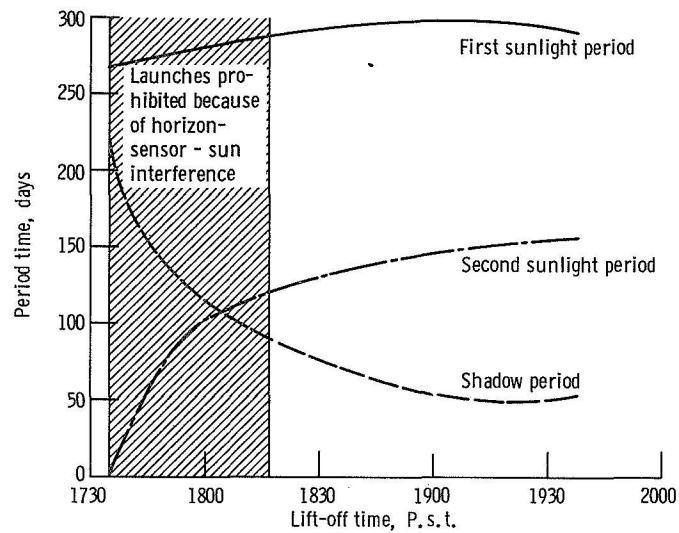


Figure 6. - Mission profile for fast precession rate and first operating orbit raising thruster.

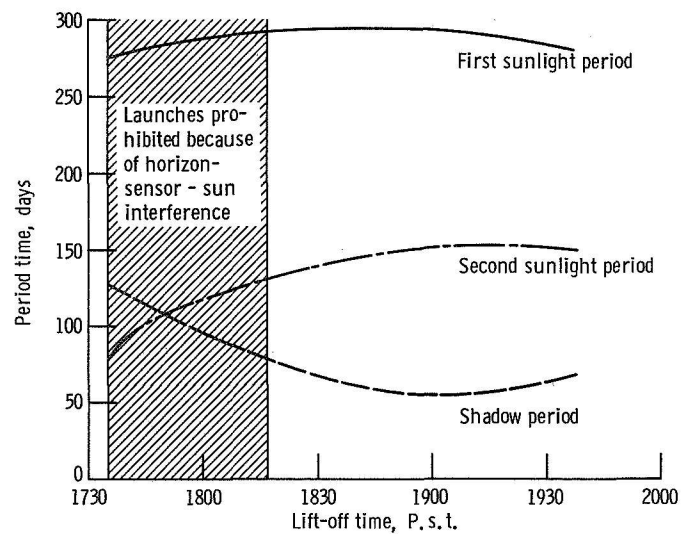


Figure 7. - Mission profile for slow precession rate and first operating orbit lowering thruster.

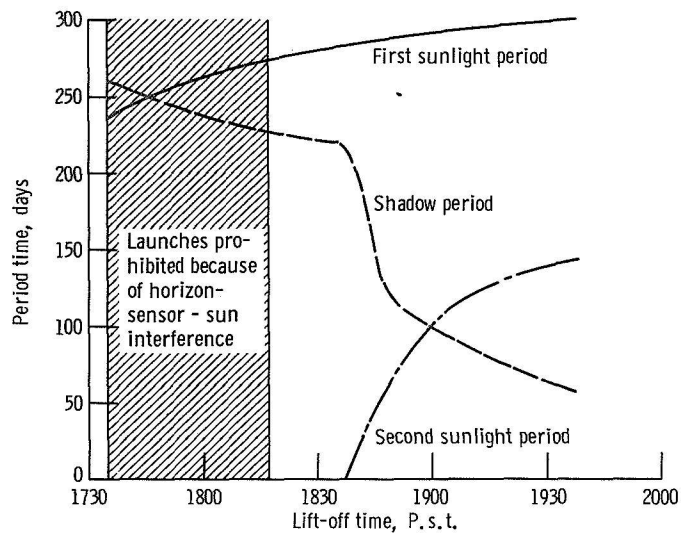


Figure 8. - Mission profile for slow precession rate and first operating orbit raising thruster.

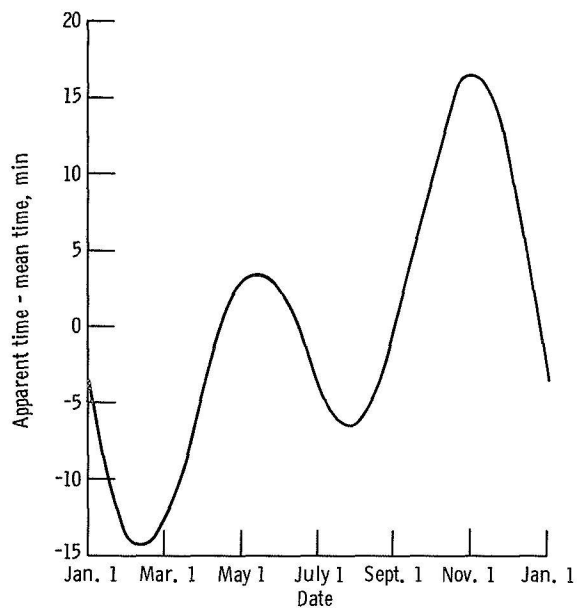


Figure 9. - Equation of time.

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